SU(2) Yang-Mills theory in 4D - review

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} , \]

where \( F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g e^{abc} A_\mu^b A_\nu^c \) is the field-strength tensor.

By construction, theory is invariant under:
1. Poincare group (translation, Lorentz boosts, rotation)
2. Internal SU(2) gauge symmetry (redundancy)

\[ A_\mu^a \rightarrow A_\mu^a - \frac{1}{f} \partial_\mu \alpha^a(x) - e^{abc} \alpha^b(x) A_\mu^c + O(\alpha^2) \]

space-time-dependent parameter.

Often, it pays to express \( A_\mu^a(x) \) as a matrix-valued field:

Write \( A_\mu = A_\mu^a T^a \leftarrow T^a = \frac{\sigma^a}{2} = \) generators of SU(2) in fundamental representation

and \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i g [A_\mu, A_\nu] \)

\[ = F_{\mu\nu}^a T^a \]

So that now the Lagrangian takes the form:

\[ \mathcal{L} = -\frac{1}{2} Tr \left[ F_{\mu\nu} F^{\mu\nu} \right] , \]

and the theory is invariant (rather, redundant) under

\[ A_\mu \rightarrow U(x) A_\mu U^\dagger(x) = \frac{i}{f} U(x) \partial_\mu U^\dagger(x) , \]

with \( U(x) = e^{i \alpha^a(x) T^a} \in \text{SU}(2) \) at each space-time point.

There are two additional accidental symmetries:

3. Scale transformations: \( \chi^\mu \rightarrow \lambda \chi^\mu \)

4. Special conformal transformations: \( \chi^\mu \rightarrow \frac{\chi^\mu + b^\mu x^2}{1 + 2 b \cdot x + b^2 x^2} \)