Lippmann-Schwinger equation for $\hat{G}(\lambda)$.

Hamiltonian: $H_0 = \frac{p^2}{2m}, \quad H = \frac{p^2}{2m} + V$

Introduce two Green's functions (Resolvent operators):

$G_0(\lambda) = (\lambda - H_0)^{-1}$ for free Hamiltonian

$G(\lambda) = (\lambda - H)^{-1}$ for interacting Hamiltonian.

Analytic structure of $G_0$ & $G$:

- $\hat{G}_0$  
- $\hat{G}$  
- $\hat{L}$

$\text{free plane wave}$  
$\text{bound states}$  
$\text{scattering states}$

Solving for $\hat{G}(\lambda)$ is just as hard as solving eigenvalue problem.

However, we can relate $\hat{G}(\lambda)$ to the known $G_0(\lambda)$ and $V$.

Use identity: $A = B + A - B$

$= B + (BB^{-1})A - B(A^{-1}A)$

$= B + B(B^{-1} - A^{-1})A$

Set $A = G(\lambda)$ and $B = G_0(\lambda)$

$A^{-1} = \lambda - H_0$

$B^{-1} = \lambda - H_0$

$G(\lambda) = G_0(\lambda) + G_0(\lambda)\left(\lambda - H_0 - \lambda + H\right)G(\lambda)$

$= G_0(\lambda) + G_0(\lambda)\left(\lambda - H_0\right)G(\lambda)$

$\hat{G}(\lambda) = \hat{G}_0(\lambda) + \hat{G}_0(\lambda)\hat{V}\hat{G}(\lambda)$

also, $\hat{G}(\lambda) = \hat{G}_0(\lambda) + \hat{G}(\lambda)\hat{V}\hat{G}_0(\lambda)$

Lippmann–Schwinger equation for $\hat{G}(\lambda)$ by interchanging $A$ & $B$. 