Properties of Born-level approximation

\[ f_{\text{B}}(\xi) = -2m \int_0^{\infty} dr \, r^2 \sin \frac{\theta r}{q} V(r) \quad \text{for spherically symmetric potentials.} \]

\[ |\xi| = \frac{q}{\xi} = \frac{2E}{m} \sin \theta/2 \]

\section{Forward Amplitude}

\[ f_{\text{B}}(\xi = 0) = -2m \int_0^{\infty} dr \, r^2 V(r) = \text{constant in } E \]

\[ \text{constant in } V(r) = \frac{1}{\sqrt{2\pi}} \text{ as } r \to \infty \]

\[ \text{in Coulomb} \]

\section{Zero Energy (q threshold)}

\[ f_{\text{B}}(\xi = 0) = \text{(same as above)} = \text{constant in } \theta \text{ (isotropic)} \]

This is true for the exact amplitude \( f(E' \to E) \) although the behavior of Born approximation in this regime is irrelevant.

Since Born approximation is unreliable at low energies.

\section{High Energy (forward peak)}

\[ E \to \text{large or } |E'\to E| \to 0 \]

\[ \Rightarrow |\xi| \to \text{large} \]

\[ \text{Integral and hence } f_{\text{B}} \text{ suppressed.} \]

\[ \text{Except at forward angles } \theta = 0 \text{ where } |\xi| = 0 \text{ (going back to case 3)} \]

\[ |f_{\text{B}}|^2 = \frac{d\sigma}{d\Omega} \] at zero energy

\[ \text{at high energy} \]

\[ \text{FORWARD PEAK} \]

\[ \text{at zero energy} \]

\[ \text{at high energy} \]

\[ \text{Forward} \]

\[ \frac{d\sigma}{d\Omega} \]

\[ \theta \]

\[ \text{becomes } \]

\[ \theta \]

\[ \text{not satisfied} \]

\[ \text{simple probabilistic analysis reveals} \]

that \( f_{\text{B}}(\xi) \) is real-valued.