Cases:

\[ \omega = \omega' \quad (\text{Elastic Scattering}) \]

1. If \( \omega \gg \left( \frac{E_{i} - E_{k}}{\gamma} \right) \) (photon frequency much higher than atomic energy differences),

\[
\frac{d\sigma}{d\Omega} \approx \frac{\alpha e^2}{m_e^2} \left( \frac{\omega}{c} \right) \left| \frac{d}{dE} E^{*} E + \frac{C}{(\gamma E)} \right|^2
\]

Can neglect denominators. [Is dipole approx still valid?]

\[ \omega = c/\lambda = \frac{2\pi}{\lambda} = 2.3 \text{ keV} \]

Atomic transition frequency \( \Omega \text{ (eV)} \)

So yes! Cool valid.

2. Low energy: Rayleigh scattering.

Why the sky is blue:

If \( \omega \ll \left( \frac{E_{i} - E_{k}}{\gamma} \right) \), use dipole form, and neglect widths and co in denominator.

\[
\frac{d\sigma}{d\Omega} = \frac{m^2}{\alpha e^2} \left( \frac{\omega^2}{c} \right) \left| \sum_n \left( \frac{\vec{e}_{k,n} \cdot \vec{E}_{n}}{E_{i} - E_{k}} + \frac{\vec{e}_{k,n} \cdot \vec{E}_{k}}{E_{k} - E_{n}} \right) \right|^2
\]

- Not suppressed by coma.
- Higher frequencies are most strongly scattered.

Among the colors of the visible spectrum:

Violet is most scattered, then blue, green ... red scattered least.

But this is also most attenuated.

Thus the red colored Sun at dawn.
Resonant scattering

(Fluorescence)

Kramers-Henneberg formula derives as $\hbar \omega \rightarrow E_B - E_i$

Damping effects not included

Model: Driven oscillator with damping.

\[ \dot{x} + \gamma \dot{x} + \omega_0^2 x = \cos \omega t \]

For long times, \( x(t) = \frac{a}{\sqrt{(a_0^2 - \omega^2)^2 + \omega_0^2 \gamma^2}} \cos (\omega t - \tan^{-1} \frac{\omega \gamma}{a_0^2 - \omega^2}) \)

On resonance when \( \omega = \omega_0 \),

\[ x(t) = \frac{a}{\omega \gamma} \cos (\omega t - 90^\circ) \]

Note: On resonance amplitude is 90° out of phase with driving force.

Applied to atomic system:

Electric component of EM field is the driving force.

The electron is like an oscillator.

The emitted EM radiation is the damping.

Expect the dependence (after external field is turned off) \( \sim e^{-t/2\tau} \), \( \tau = \text{lifetime of state } n \)

Maintenance:

Need to draw internal atomic line propagator:

\[ A_n \]

\[ \rightarrow \text{ propagator methods} \]