Origin of signature

\[ *-\text{channel}: \ 12 \rightarrow 34 \]

\[ \overrightarrow{1} \rightarrow e \Theta_{12} \overrightarrow{3} \]
\[ \Theta_{12} \equiv \text{angular variable in } t-\text{channel} \]
\[ \equiv \text{energy variable in } s-\text{channel} \]
\[ \text{take } \Theta_{12} \rightarrow \infty \text{ for Regge limit} \]

\[ u-\text{channel}: \ \overline{42} \rightarrow 3 \overline{5} \]

\[ \overrightarrow{4} \rightarrow e \Theta \overrightarrow{5} \]
\[ \Theta_{34} \equiv \text{angular variable in } u-\text{channel} \]
\[ \equiv s_{34} = s_{12} \text{ energy variable in } s-\text{channel} \]
\[ \text{Take } \Theta_{34} \rightarrow \infty \text{ for Regge limit} \]

The two contributions have a phase difference of \((-1)^l = e^{i\pi l}
\]

\[ \text{bad } l \rightarrow \text{zoo behavior} \]
\[ \Rightarrow \text{introduce signature} \]
More on the Regge signature.

The origin of signature is as follows — it comes ultimately from spin-statistics & indistinguishability — leads to exchange.

Signature comes from left-hand cut in $A(t,w)$ sc. amp.

Left-hand cut comes from exchange graphs due to indistinguishability of particles.

In potential scattering, left-hand cut is absent.

In continuing the partial wave amplitudes to non-integer $l$, the question of uniqueness arises. Important because different continuants would lead to different Regge terms.

$$A_{\text{new}}(s) = A_{\text{old}}(s) + f(l)$$

$\uparrow$ $\uparrow$ $\uparrow$

different continuant

chosen

continuation

difference function.

Here, Carlson’s theorem is needed.

Let $f(l)$ satisfy

(i) Regular in $\text{Re}(l) > L$ (no singularities past $L$)

(ii) $f(l) < e^{M(l)}$ (exponential bounded)

for $\text{Re}(l) > L$

(iii) $f(l) = 0$ for an infinite sequence (in our case $l \in \mathbb{Z}$)

Then $f(l) = 0$ for all $l$.

Therefore, we need a continuant satisfying

0. Exponential boundedness.

1. Regularity for $\text{Re}(l) > L$ — postulate of "Maximal analyticity of the second kind."