Dual diagrams and overlapping channels

Scattering process:

Dual diagram:

A polygon displaying the conservation of linear momentum.

Channel energy: Represented by a diagonal of the polygon.

\[ S_{24} = (p_2 + p_3 + p_4)^2 \equiv (p_5 + p_6 + p_2)^2 = S_{56} \]

Overlapping channel: Channels whose diagonals cross in the dual diagram. Also called dual channels - they represent the angular variable in one another's channel.

\[ S_{23} \text{ & } S_{34} \text{ are overlapping} \]
\[ S_{24} \text{ & } S_{56} \text{ are not overlapping} \]
\[ S_{55} \text{ & } S_{54} \text{ are not overlapping} \]
\[ (S_{54} \text{ is subenergy of } S_{55}) \]
Because dual channel energies (like $t$) represent the angular variable of a direct channel (like $s$-channel),

No physical amplitude can contain a simultaneous (double, or higher order-) pole in overlapping channel energies (like $s \& t$).

**Reason:**
Suppose an amplitude had a pole at $s = m^2$. By factorization,

$$A(s, t, \ldots) \sim \frac{-g^2 P_j(t)}{s - m^2} + \ldots$$

The residue function $P_j(t)$ provides information about the spin:
- it is a polynomial in $t$ of order $J$.

If $P_j(t)$ itself had a pole within the physical region,

$$P_j(t) \approx \frac{1}{(t - t_0)}$$

it would no longer be a polynomial in $t$

$\Rightarrow$ infinite number of particles of all possible spins.

$$A(s, t) \sim \frac{1}{s} + \frac{1}{t} \text{ OK!}$$

$$A(s, t) \sim \frac{1}{st} \text{ NOT OK}$$

Feynman diagrams automatically forbid "overlapping poles".

Impossible to draw a graph with overlapping poles.

The residue of this $s$-channel pole must be a polynomial.