Gauge transformation of vector superfield:

Let: \[ V(x, \theta, \theta^\dagger) \] be a vector superfield for U(1) gauge symmetry.

Consider the transformation:

\[ V \rightarrow V - i(\Omega, -\Omega^\dagger) \]

where \( \Omega \) = space-time dependent chiral superfield
gauge transformation parameter:

\[
\Omega_\mu = \beta + \theta_\mu \partial \omega + \theta \partial s + \theta_\mu \theta^\dagger (-\bar{\omega} \beta) + \theta^\dagger \left[ \frac{i}{4 \omega} \bar{\omega} \partial \bar{\omega} \mu \right] \theta \bar{\theta}^\dagger + \theta \theta^\dagger \theta^\dagger \left[ \frac{i}{4 \omega} \bar{\omega} \partial \bar{\omega} \mu \right] \theta \bar{\theta}^\dagger + \theta \theta^\dagger \theta^\dagger \left[ \frac{i}{4 \omega} \bar{\omega} \partial \bar{\omega} \mu \right] \theta \bar{\theta}^\dagger
\]

so that, in components:

\[ \phi \rightarrow \phi - i(\beta - \beta^\dagger) \]

\[ \chi \rightarrow \chi + \sqrt{2} i \omega \]

\[ \chi^\dagger \rightarrow \chi^\dagger - 2 i \omega^\dagger \]

\[ f \rightarrow f + s \]

\[ f^\dagger \rightarrow f^\dagger + s^\dagger \]

\[ A_\mu \rightarrow A_\mu - i \partial_\mu (\beta + \beta^\dagger) \]

\[ (\lambda^\dagger - \frac{i}{2} \bar{\sigma}^\mu \partial_\mu \lambda) \rightarrow \lambda^\dagger - \frac{i}{2} \bar{\sigma}^\mu \partial_\mu \lambda + \frac{i}{2} \partial_\mu \bar{\sigma}^\mu \lambda \]

\[ \Rightarrow \lambda^\dagger \rightarrow \lambda^\dagger \quad \text{(invariant, by design)} \]

\[ \lambda \rightarrow \lambda \quad \text{(invariant, by design)} \]

\[ \frac{1}{2} D^2 \phi - \frac{\alpha}{4} \phi^2 \rightarrow \frac{1}{2} D^2 \phi - \frac{1}{4} \phi^2 (\beta^\dagger - \beta) \]

\[ D \rightarrow D \quad \text{(invariant, by design)} \]
Super-gauge transformation parameters represents 8 degrees of freedom:

\[
Re(\beta), \quad Im(\beta), \quad \omega, \quad \xi
\]

\[
\begin{array}{c}
1 + 1 + 4 + 2 = 2
\end{array}
\]

Use these to eliminate
\[
\phi, \quad \chi, \quad f \text{ in vector superfield:}
\]

\[
\begin{align*}
V_{\mu\alpha}(x; \theta, \theta^t) &= \theta \sigma^\mu \theta^\dagger \overline{A}_\mu(x) + \theta^t \lambda^\dagger(x) \theta \theta + \theta^t \theta^\dagger \theta \lambda(x) + \frac{1}{2} \theta^t \theta^\dagger \theta \theta \overline{D}(x).
\end{align*}
\]

leaving \( A_\mu \rightarrow A_\mu - i \partial_\mu 2 \operatorname{Re}[\beta] \) as the residual
gauge transformation.

Notes:

- Under a SUSY transformation, the vector superfield no longer satisfies Wess-Zumino gauge.

\[
\Rightarrow \text{Wess-Zumino gauge is not a SUSY-invariant gauge.}
\]

(like how Coulomb or Axial gauges are not Lorentz invariant).

- In terms of \( y^\mu = x^\mu - i \theta \sigma^\mu \theta^\dagger \), vector superfield in the
Wess-Zumino gauge is not very different:

\[
A_\mu(x) = A_\mu(y + i \theta \sigma^\mu \theta^\dagger)
\]

(all other components give higher order in \( \theta \) & \( \theta^t \))

and use \( (\theta \sigma^\mu \theta^\dagger)(\theta \sigma^\nu \theta^\dagger) = \frac{1}{2} g^{\mu\nu}(\theta \theta^t)(\theta \theta^t) \) calculated previously:

\[
\begin{align*}
V_{\mu\alpha}(y; \theta, \theta^t) &= \theta \sigma^\mu \theta^\dagger \overline{A}_\mu(y) + \theta^t \lambda^\dagger(y) \theta \theta + \theta^t \theta^\dagger \theta \lambda(y)
+ \frac{1}{2} \theta^t \theta^\dagger \theta \theta [D(y) + i \partial.A(y)]
\end{align*}
\]
Properties of Vector superfields

Vector superfields may be written in terms of chiral superfields.

\[ \Phi = \text{left-chiral superfield} \]
\[ \Phi^* = \text{right-chiral superfield} \]

Then \( \Phi + \Phi^* \), \( i(\Phi - \Phi^*) \), \( \Phi \Phi^* \) are vector superfields.

\[ \text{"Real part"} \]
\[ \text{"Imaginary part"} \]
\[ \text{"Modulus"} \]