Building a supersymmetric Lagrangian

- Two types of terms may enter into the action:
  (named after components of the superfield transforming as a total derivative)

**F-term:**

\[
\Phi \left( \right)_F = \int d^2 \theta \Phi \left|_{\theta=0} = F(\chi) \quad \text{susy} \quad -i \sqrt{2} \alpha^+ \bar{\sigma}^\mu \partial_\mu \chi(\chi) \right.
\]

c.c.

\[
[\Phi]_F = \int d^2 \theta^+ \Phi^+ \left|_{\theta^+=0} = F^*(\chi) \quad \text{susy} \quad i \sqrt{2} \partial_\mu \chi^+(\chi) \bar{\sigma}^\mu \alpha \right.
\]

(total derivative)

Since the action is real (to generate a Hermitian Hamiltonian),
both terms must be included:

\[
S = \int d^4 x \left( [\Phi]_F + \text{c.c.} \right) \quad (\ast)
\]

**D-term:**

\[
[V]_D = \int d^2 \theta d^2 \theta^+ V = D(\chi) \quad \text{susy} \quad -i \left( \lambda^a \bar{\gamma}^\mu \gamma^\mu \chi + \alpha^+ \bar{\sigma}^\mu \gamma^\mu \right).
\]

Since the vector superfield is real (by construction),
D-terms can be added as is:

\[
S = \int d^4 x \left[ V \right]_D \quad (\ast\ast)
\]

Contributions (\ast) and (\ast\ast) are the only types of terms that may be added to the action in a (four-dimensional) supersymmetric theory.