Imposing Majorana conditions on u × ν spinors: \( C = i\gamma^5 \gamma^0 \)

(1) \( \bar{\psi}^c = C \bar{\psi}^\gamma \)

\[
= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} \sum_S \left[ \hat{\alpha}_{\bar{p}_\downarrow} \frac{c_u^T}{v} e^{i\theta \cdot x} + \hat{\alpha}_{\bar{p}_\uparrow} \frac{c_\nu^T}{u} e^{-i\theta \cdot x} \right]
\]

\( = \bar{\psi} \) (Majorana condition)

\[ \therefore \begin{align*}
C u^T &= v \\
\text{and} \quad C \nu^T &= u
\end{align*} \]

or

\[ u^c = v \\
\nu^c = u \]

(2) \( \bar{\psi}^c = -e^{i\theta^T} \)

\[
= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} \sum_S \left[ \hat{\alpha}_{p_\downarrow} \left( -u^T C^{-1} \right) e^{-i\theta \cdot x} + \hat{\alpha}_{p_\uparrow} \left( -\nu^T C^{-1} \right) e^{i\theta \cdot x} \right]
\]

\( = \bar{\psi} \) (Majorana condition)

\[ \therefore -u^T C^{-1} = \bar{v} \quad \text{and} \quad \nu^T C^{-1} = \bar{u} \]

or

\[ u^T C^{-1} = -\bar{v} \\
\nu^T C^{-1} = \bar{u} \\
u^c = u \]

These identities are valid for Dirac spinors. Only difference between
Dirac and Majorana cases is the Fourier amplitude: \( b, d \rightarrow a \).