Propagators

Time-ordered Green function:

\[
\langle 0 | T \psi_\alpha(x) \bar{\psi}_\beta(x') | 0 \rangle = \Theta(x_0 - x'_0) \frac{\langle 0 | \psi_\alpha(x) \bar{\psi}_\beta(x') | 0 \rangle}{G^>} + \Theta(x'_0 - x_0) \frac{\langle 0 | \bar{\psi}_\beta(x') \psi_\alpha(x) | 0 \rangle}{G^<}
\]

\[
G^>(x-x') = \langle 0 | \bar{\psi}_\beta(x') \psi_\alpha(x) | 0 \rangle = \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \frac{1}{2\sqrt{p_1 \cdot p_2}} \sum_{s,s'} \langle 0 | \left( \delta_{\tilde{p}_3,s} \bar{u}_\alpha^1(p_1) e^{-ip_1 \cdot x} + \delta_{\tilde{p}_3,s} u_\alpha^1(p_2) e^{ip_2 \cdot x} \right)
\times \left( \delta_{\tilde{p}_4,s'} \bar{u}_\beta^2(p_2') e^{ip_2' \cdot x'} + \delta_{\tilde{p}_4,s'} u_\beta^2(p_1') e^{-ip_1' \cdot x'} \right) | 0 \rangle
\]

Only one term contributes, so computation proceeds exactly as in Dirac case.

\[
\therefore \langle 0 | T \psi_\alpha(x) \bar{\psi}_\beta(x') | 0 \rangle = \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x-x')} \frac{i(p+m)\gamma_\mu}{p^2 - m^2 + i\epsilon}
\]

That the Majorana spinor has half as many degrees of freedom as a Dirac fermion is encoded in the factor $\frac{1}{2}$ in the canonically normalized Majorana kinetic term.

Two more Majorana-type propagators:

\[
\psi_c = \psi = C \bar{\psi}^T \quad \Rightarrow \quad \bar{\psi}^T = C^{-1} \psi
\]

\[
\bar{\psi}_c = \bar{\psi} = -\psi^T C^{-1} \quad \Rightarrow \quad \psi^T = -\bar{\psi} C
\]

Then,

\[
\langle 0 | T \psi_\alpha(x) \bar{\psi}_\beta(x')^T | 0 \rangle = \langle 0 | T \psi_\alpha(x) \bar{\psi}_\beta(x') | 0 \rangle (-C)_{\beta'\gamma}
\]

\[
\langle 0 | T \bar{\psi}_\alpha(x)^T \bar{\psi}_\gamma(x') | 0 \rangle = (C^{-1})_{\gamma\beta} \langle 0 | T \bar{\psi}_\beta(x) \bar{\psi}_\gamma(x') | 0 \rangle
\]