Atypical completeness relations,
\((u, v)\) cross relations

\[ \sum_{S=\pm\frac{1}{2}} U_{\text{Dirac}}^{[3]}(p^-) \cdot V_{\text{Dirac}}^{[3]}(p^+) \]

\[ = \sum_{S} \left( U_{\alpha}^{[3]}(p^-) \cdot V_{\beta}^{[3]}(p^+) \right) \cdot \left( \begin{array}{c} \gamma_\alpha \gamma_\beta \\ \gamma_\beta \gamma_\alpha \end{array} \right) \]

\[ = \sum_{S} \left( \begin{array}{c} \gamma_\alpha \gamma_\beta \\ \gamma_\beta \gamma_\alpha \end{array} \right) \cdot \left( \begin{array}{c} \gamma_\alpha \gamma_\beta \\ \gamma_\beta \gamma_\alpha \end{array} \right) \]

\[ = \begin{pmatrix} m \gamma_\alpha \gamma_\beta p_\alpha p_\beta \\ p_\alpha p_\beta - m^2 \end{pmatrix} \]

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\[ = (p^2 + m^2) \gamma_\alpha \gamma_\beta p_\alpha p_\beta \]

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Similarly,

\[ \sum_{S=\pm\frac{1}{2}} U_{\text{Dirac}}^{[3]}(p^-) \cdot V_{\text{Dirac}}^{[3]}(p^+) = \frac{i \gamma^2 \gamma^0 (p^2 + m^2)}{C} \]