Full Propagators - general class of linear gauges

Define: \[ i\Pi^{\mu\nu}(p) = \frac{p^2}{-m^2} = i\left(P_{\alpha}A_{\mu\nu} + P_{\beta}B_{\mu\nu} + P_{\gamma}C_{\mu\nu} + P_{\delta}E_{\mu\nu}\right) \] [1]

Then, the full propagator is

\[ \tilde{D}^{\mu\nu}_{\text{Full}}(p^2) = \tilde{D}^{\mu\nu} + \tilde{D}^{\mu\rho}_{\mu\nu} \approx \tilde{D}^{\mu\nu} + \tilde{D}^{\mu\rho}_{\mu\nu} \approx \tilde{D}^{\mu\nu} + \tilde{D}^{\mu\rho}_{\mu\nu} \approx \tilde{D}^{\mu\nu} + \tilde{D}^{\mu\rho}_{\mu\nu} \approx \tilde{D}^{\mu\nu} + \tilde{D}^{\mu\rho}_{\mu\nu} + \cdots \]

\[ = \tilde{D}^{\mu\nu} + \tilde{D}^{\mu\rho}_{\mu\nu} \approx \tilde{D}^{\mu\nu} + \tilde{D}^{\mu\rho}_{\mu\nu} \approx \tilde{D}^{\mu\nu} + \tilde{D}^{\mu\rho}_{\mu\nu} \approx \tilde{D}^{\mu\nu} + \tilde{D}^{\mu\rho}_{\mu\nu} + \cdots \]

\[ = \tilde{D}^{\mu\rho}_{\mu\nu} \sum_{n=0}^{\infty} (i\Pi^{\rho\nu})^{-n} \]

Geometric series

\[ = \tilde{D}^{\mu\rho}_{\mu\nu} (1 - i\Pi^{\rho\nu})^{-1} \]

\[ \tilde{D}^{\mu\nu}_{\text{Full}} = (1 - i\Pi^{\rho\nu})^{-1} \tilde{D}^{\mu\rho}_{\mu\nu} \]

\[ \tilde{D}^{\mu\nu}_{\text{Full}} = \tilde{D}^{\mu\nu} - i\Pi^{\mu\nu} \] [2]

Now, \[ \tilde{D}^{\mu\nu} = -i\hat{\Theta}^{\mu\nu} \] (by definition)

\[ = -i(p^2 A^{\mu\nu} + i(p^2 + \frac{4}{3}m^2)B^{\mu\nu} + i\left(\frac{p^4}{2} - \pi^2\right)C^{\mu\nu} + i\left(\frac{p^4}{3} - \pi^2\right)E^{\mu\nu} \] [3]


\[ \tilde{D}^{\mu\nu}_{\text{Full}} = \frac{i(p^2 - \pi^2)A^{\mu\nu} + i(p^2 + \frac{4}{3}m^2 - \pi^2)B^{\mu\nu} + i\left(\frac{p^4}{2} - \pi^2\right)C^{\mu\nu} + i\left(\frac{p^4}{3} - \pi^2\right)E^{\mu\nu}}{a} \]

Inverse this to obtain full propagator - follow precisely the same steps used to

invent \[ \hat{\Theta}^{\mu\nu} \]: define \[ a, b, c, \text{ & } e \] in the same way - indicated above.

Ansatz: \[ \tilde{D}^{\mu\nu}_{\text{Full}}(p^2) = a' A^{\mu\nu} + b' B^{\mu\nu} + c' C^{\mu\nu} + e' E^{\mu\nu} \]

Solve for \[ a', b', c', \text{ & } e' \] (known from inverting \[ \hat{\Theta}^{\mu\nu} \])
Solution (borrowing previous results):

\[ a' = \frac{1}{\alpha} = \frac{-2}{p^2 - \Pi_A} \]

\[ b' = \frac{e}{b - \frac{c^2}{e} p^2 n^2} = \frac{1}{r(p)} \]

\[ c' = \frac{c}{b - \frac{c^2}{e} p^2 n^2} = \frac{1}{r(p)} \]

\[ c' = \frac{b}{b - \frac{c^2}{e} p^2 n^2} = \frac{1}{r(p)} \]

where \( r(p) = \frac{1}{b} \left( b - \frac{c^2}{e} p^2 n^2 \right) = -p^2 - \frac{1}{\epsilon} n^2 + \frac{1}{\epsilon} \left( \frac{p_f^2}{p^2} - \Pi_T \right) + \frac{1}{\epsilon} \left( \frac{p_f^2}{p^2} - \Pi_F \right) \]

\[ \mathcal{S}_{\text{Full}}(p^2) = \frac{-i}{p^2 - \Pi_A} A^{\mu \nu} + \frac{i}{r(p)} B^{\mu \nu} - \frac{i}{r(p)} (p_f - \frac{1}{\epsilon} p^2 \Pi_T) C^{\mu \nu} \]

\[ + \frac{i}{r(p)} (p_f)^2 \left( \frac{1}{p^2 - \frac{1}{\epsilon} \Pi_T} - \frac{1}{p^2 - \frac{1}{\epsilon} \Pi_F} \right) E^{\mu \nu} \]

In class of covariant gauges: \( f^M = -ip^M \rightarrow p^M \) (integ. by parts)

\[ \Rightarrow \quad n^M = 0 \quad B^{\mu \nu} = C^{\mu \nu} = 0. \]

\[ \Pi_B(p^2) = \Pi_C(p^2) = 0 \]

\[ \Rightarrow \quad r(p) = -p^2. \]

So coeff. of \( E^{\mu \nu} \) term = \(-\frac{i}{p^2 - 1 - \frac{5}{\epsilon} \Pi_F} = \frac{-\xi i}{p^2 - \frac{5}{\epsilon} \Pi_F} \)

Conventionally, \( \Pi_A \equiv \Pi_T \) (transverse pol. function)

\( \Pi_F \equiv \Pi_L \) (longitudinal pol. function)

\[ \Rightarrow \quad \mathcal{S}_{\text{Full}}(p^2) = \frac{-i}{p^2 - \Pi_f(p^2)} \left( g^{\mu \nu} - \frac{p^M p_N}{p^2} \right) - \frac{\xi i}{p^2 - \frac{5}{\epsilon} \Pi_L(p^2)} \frac{p^M p_N}{p^2} \]

In abelian gauge theories,

\( \text{Ward identity: } \Rightarrow \Pi_T(p^2 = 0) = 0 \)

\( \Pi_L(p^2) = 0 \)
In \( \mathbb{R}^\infty \) gauge: (Spontaneously broken gauge theories)

\[
\partial_a A^a + \xi \left[ j^a \langle a \rangle \right]_i \gamma^a = 0 \quad \Rightarrow \quad f^\mu = -i p^\mu \rightarrow p^\mu \quad (\text{only by path})
\]

\[
\Rightarrow \quad \gamma^\mu = 0 \quad B^{\mu \nu} = C^{\mu \nu} = 0
\]

\[
\Pi_B(p^2) = \Pi_c(p^2) = 0
\]

From:

\[
L_{\text{quad}} = \frac{1}{2} \ A^a (\partial^2 g^{\mu \nu} \delta^{ab} - (1 - \frac{i}{\xi}) \ \partial^\mu \gamma^a \delta^{ab} + (m_a^2)_{ab} g^{\mu \nu}) A^b,
\]

\[
\Rightarrow \quad \widetilde{D}^{-1}_{\mu \nu} = i \left( p^2 \delta^{ab} - (m_a^2)_{ab} \right) \left( g_{\mu \nu} - \frac{P_{\mu \nu}}{p^2} \right) + i \left( \frac{p^2 - (m_a^2)_{ab}}{p^2} \right) \frac{P_{\mu \nu}}{E_{\nu}} \quad \text{see deriving R_\xi propagator.}
\]

Add \(-i \Pi_{\mu \nu} = -i \left( \Pi_B(p^2) A_{\mu \nu} + \Pi_c(p^2) E_{\mu \nu} \right)\)

\[
\Rightarrow \quad \widetilde{D}^{-1}_{\text{Full} \mu \nu} = i \left( p^2 \delta^{ab} - (m_a^2)_{ab} - \Pi_B(p^2) \right) A_{\mu \nu} + i \left( \frac{p^2 - (m_a^2)_{ab} - \Pi_E(p^2)}{p^2} \right) E_{\mu \nu}
\]

Can be easily inverted since \( A_{\mu \nu} \) and \( E_{\mu \nu} \) are orthogonal projectors:

\[
\widetilde{D}_{\text{Full} \mu \nu} = \frac{-i}{p^2 - (m_a^2)_{ab} - \Pi_B(p^2)} \left( g_{\mu \nu} - \frac{P_{\mu \nu}}{p^2} \right) + \frac{-i \xi}{p^2 - \xi (m_a^2)_{ab} - \xi \Pi_E(p^2)} \frac{P_{\mu \nu}}{p^2}
\]

Conventionally,

\[
\Pi_B = \Pi_T,
\]

\[
\Pi_E = \Pi_L
\]

Gauge invariance \( \Rightarrow \Pi_T(0) = \Pi_L(0) \) (to be checked?)
Projectors for $T^\mu_\nu$, and pole masses of gauge bosons.

In covariant fermi (e.g. $\theta\cdot A = 0$ for no Higgs mechanism),
and in $\tilde{R}$ gauges (with Higgs mechanism),

$$\tau T^\mu_\nu(p^2) = \tau T_T(p^2) \left( g^{\mu\nu} - \frac{p^{\mu} p^{\nu}}{p^2} \right) + \tau T_L(p^2) \frac{p^{\mu} p^{\nu}}{p^2}$$

$$\tau B^{\mu\nu}_I(p^2) = \frac{-i}{p^2 - m_A^2 - \tau T_T(p^2) \left( g^{\mu\nu} - \frac{p^{\mu} p^{\nu}}{p^2} \right)} + \frac{-i \tilde{\tau} \frac{3}{2} \tau T_L(p^2)}{p^2 - \frac{3}{2} (m_A^2 + \tau T_L(p^2))} \frac{p^{\mu} p^{\nu}}{p^2}$$

To extract $T_T(p^2)$ and $T_L(p^2)$ from a rank-2 tensor,
contract with projectors:

$$T^\rho_\sigma = \frac{1}{d-1} \left( \delta^\rho_\sigma - \frac{p^\rho p_\sigma}{p^2} \right)$$  \hspace{1cm} \text{transverse projector}

$$L^\rho_\sigma = \frac{p^\rho p_\sigma}{p^2}$$  \hspace{1cm} \text{longitudinal projector}

and take trace:

\[ \text{TRANSVERSE} \hspace{1cm} \text{LONGITUDINAL} \]

$$\tau T^\rho_\mu(p^2) T^\rho_\mu = \tau T_T(p^2)$$  \hspace{1cm} $$\tau T^\rho_\mu(p^2) L^\rho_\mu = \tau T_L(p^2)$$

\[ \begin{aligned}
T^\rho_\mu T^\nu_\mu &= \frac{-i}{\tau T_T(p^2)} \frac{T^\rho_\mu}{\tau T_T(p^2)} - \frac{i}{\tau T_L(p^2)} \frac{T^\rho_\mu}{\tau T_L(p^2)} \\
\tau T_T(p^2) \tau T_L(p^2) &= \tau T_T(p^2) L^\nu_\mu = \tau T_L(p^2)
\end{aligned} \]

Pole masses

The position of the pole — for 3-transverse directions — is:

$$p^2 - \tau T_T(p^2) \bigg|_{p^2 = m^2_{pole}} = 0$$  \hspace{1cm} $$m_A^2 - \tau T_T(p^2) \bigg|_{p^2 = m^2_{pole}} = 0$$  \hspace{1cm} $$Z/W$$ gauge boson: